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Original paper

Developing a fuzzy programming model for improving outpatient appointment scheduling

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Abstract

Appointment scheduling for outpatient services is a challenge in the healthcare sector. For addressing this challenge, most studies assumed that patients unpunctuality and the duration of service have constant values or a specific probability distribution function. Consequently, there is a research gap to consider the uncertainty of both patients unpunctuality and the duration of service in terms of fuzzy sets. Therefore, this research aims to consider fuzzy values for both unpunctuality and duration of service have to improve an outpatient appointment scheduling (the time interval between two patients) in a referral clinic with the objective of reducing the total weight of waiting time, idle time, and overtime. Four different fuzzy linear programming models and 36 scenarios have been developed based on the show, no-show of patients, single-book, and double-book by using GAMS software. These four models are as follows: (1) probability of no-show equal to zero, (2) probability of no-show equal to 20%, (3) probability of no-show equal to zero and with double-book factor, and (4) probability of no-show equal to 20% and with double-book factor. The results of the first, second, third, and fourth models, respectively, present the scenarios considering 10, 5, 15, and 15 minutes for the time interval between two patients that have the minimum total weight of patient waiting times, physician idle times, and physician overtime. By considering these findings, the investigated referral clinic can improve its appointment systems performance. Moreover, other similar clinics can apply the proposed model for improving their appointment systems' performance.

Keywords: Appointment scheduling, fuzzy programming, unpunctuality, no-show, healthcare.

1 Introduction

In recent years, healthcare analysis has been a common issue due to the large volume of data and its impact on people's quality of life [26]. One of the global problems is the rising demand for outpatient services over the supply of physicians for healthcare. Besides, the inefficiency of the appointment system reduces provider productivity and timely access to care [26].

Van Bussel et al. [27] presented this issue by evaluating the demand, capacity, and access time of the outpatient clinic using a structured sixstep method. For this reason, accurate scheduling of patients appointment requests has been a significant subject for providing healthcare service and is important for the efficiency of the appointment system [12]. The purpose of appointment scheduling in health centers such as referral clinics is to find an exact schedule that enables the physicians to visit the maximum number of patients during a certain time frame while minimizing the patients waiting time [2].

In the appointment scheduling problem, there are many factors such as no-show [7], the unpunctuality of patients [10], double-book [4], overbook [16] and service time [3]. Several papers consider that patients are punctual (e.g.,

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[6, 17, 21, 28]). While the unpunctuality of patients is a stochastic factor that disturbs the appointment scheduling. Despite the importance of this factor, it has aroused only a few concerns. Another important factor in several papers is no-show (e.g., [18, 26, 32, 33]). This factor disturbs the appointment scheduling similar to the unpunctuality factor. In most papers, fixed or variable service time with a specified distribution such as normal or lognormal is considered. For example, in the papers of Dogru and Melouk [3], Jiang et al. [10], Zacharias and Pinedo [32], Erdogan and Denton [4], the distribution of lognormal service time is considered.

However, uncertain parameters do not have to follow the statistical distribution [20]. Therefore, while stochastic programming might not be suitable for all optimization problems under uncertainty. Fuzzy mathematical programming as a practical methodology can address this kind of optimization problem.

Therefore, this research aims to improve the outpatient appointment scheduling (the time interval between two patients) by minimizing the weighted sum of patients' waiting time, overtime, and idle time in a referral clinic. Besides, in this paper, both unpunctuality and duration of service factors are considered to have fuzzy values.

The remainder of this paper is organized as follows. In Section 2, a review of related literature is presented. In section 3, the mathematical and fuzzy optimization models are developed after discussing the assumptions and notations. In section 4, the numerical results are presented in a case study. Finally, sections 5 and 6 describe the discussion and conclusions of this research, respectively.

2 Literature review

Appointment scheduling is one of the important aspects of patient flow management [25]. Therefore, an effective appointment scheduling system is important for referral clinic performance and patient satisfaction [31]. In this section, a review of past research is reported.

Green and Savin [7] researched to reduce delays for medical appointments with a queuing approach. They considered two factors including no-show and preferences. The results show that queuing models are useful for identifying patient panel sizes for medical practices. In 2008, a stochastic overbooking model was formulated and an appointment scheduling policy for referral clinics was developed [21]. In their study, each calling patient has a no-show probability, and the overbooking policy is used to offset this problem. In their scheduling objectives, they considered patient waiting time, overtime, and patient revenue.

Robinson and Chen [24] compared two types of appointment scheduling including traditional policy and open-access. The results showed that the open-access policy is significantly better than the traditional policy in terms of the weighted average of patients' waiting time, the doctors idle time, and the doctors overtime, except when patient waiting time or probability of no-shows are low.

Erdogan and Denton [4] also investigated and solved two new models of stochastic linear programming for the appointment scheduling problem. In their study, the length of service and the number of patients in a single day were considered to have uncertain values. The goal of Klassen and Yoogalingam's research [4] was to design appointment systems to reduce patients waiting time while maintaining high-level physician utilization. In their study, simulation-optimization was applied by considering the total cost of patients waiting time, and the doctors idle time with auxiliary criteria for overtime, idle time, and waiting time.

A study on electronic appointment booking systems was conducted by Feldman et al. [6]. They aimed to maximize the expected net profit per day. A dynamic model has been developed based on a static model for checking appointment scheduling. Besides, they proposed a heuristic solution method.

Zacharias and Pinedo [32] investigated an overbooking model for scheduling entry in the US medical center under no-show behavior with different probabilities and weights. Their goal was to allocate patients to time slots by minimizing the weighted sum of patients' waiting time and the doctors idle time and overtime.

A sequential appointment scheduling method by Yan and Tang [31] in China was proposed to balance the benefits of the clinic and patients satisfaction with general patient choice. They aimed to maximize the profit of the referral clinic over a session-day with the profit of deducting patient waiting time, idle time, and overtime from the total revenue received from patient admissions.

Another research was conducted for online appointment schedule and sequencing by Erdogan et al. [5] in the US. Their objective was to minimize the average weight of patients waiting time and staffs overtime depending on the length of the session. To address this objective, they developed a stochastic integer-programming model. Lin et al. [15] also analyzed integrated resource allocation and the scheduling problem for tactical and operational planning.

There has been a great deal of literature on outpatient scheduling considering the factor of punctuality, while unpunctuality is an uncertain factor that may cause irregular entry of patients to the clinic. Jiang et al. [10] presented the problem of appointment scheduling by developing a stochastic scheduling model considering the unpunctuality of patients in China. Luo et al. [18] analyzed appointment scheduling for an outpatient department in West China Hospital. In their study, they used an M / M / 1 / N queuing model to determine the optimal scheduling window to minimize the total daily cost, considering the environmental conditions of no-show. Zhu et al. [34] investigated appointment scheduling policies with consideration of patient unpunctuality. They found that patient unpunctuality harms the appointment scheduling system. Zhu et al. [34] presented a simulation model for improving the performance of the appointment scheduling system.

Deceuninck et al. [2] also studied appointment scheduling strategies for outpatient services. They evaluated the impact of unpunctuality, no-show, and physician delay on waiting time, overtime, and idle time using stochastic programming. Dogru and Melouk [3] developed an adaptive appointment schedule (AAS) for the primary care setting in the United States in 2018. They used a simulation optimization approach to provide optimal scheduling from the perspective of both patient and medical operations. The purpose of their study was to minimize the expected costs for direct patient waiting time, indirect patient waiting time, physician idle time, and physician overtime.

Another study was conducted in the USA in 2018 to optimize the outpatient appointment system for patient satisfaction and resource utilization by Srinivas and Ravindran [26]. In their research, the average patient expectations and the average number of patients who are not able to get an appointment for the day under consideration are applied as performance coefficients to measure patient satisfaction. Several machine learning algorithms such as artificial neural network and random forest have been used to predict patient no-show.

Finally, different appointment scheduling rules that provide predictions of the best machine learning algorithm are discussed and evaluated using simulations. They discussed that the proposed scheduling rules for the studied clinic show the best performance. Li et al. [14] conducted a study to maximize the satisfaction level of the outpatient department during the shift period considering patient preferences in China.

Internet access to health care has paved the way for using an online booking system for visiting physicians. Accordingly, Samadbeik et al. [25] conducted a study to evaluate the online outpatient booking system in Iranian hospitals in 2018. Their results showed that only 13.03% of Iranian hospitals have an active online booking system. In another research, Wang et al. [28] developed a dynamic programming (DP) model for scheduling dynamic appointments scheduling with patient preferences. Their objective was to maximize the expected performance of the system. Monahan and Fabbri [19] conducted a study to evaluate clinic performance and the potential recovery of canceled appointments in a US cardiovascular department. A study by Xiao et al. [30] at the Nuclear Medicine Department at West China Hospital was conducted to achieve a scientifically acceptable schedule.

Moreno and M. Blanco also [20] used an integer linear programming (ILP) model for the patient appointment scheduling problem. Their main aims were to minimize the admission dates (with high priority) and the length of stay. In their study, they considered uncertainty for several parameters especially in the availability of clinical services in every slot. To deal with this uncertainty, they used a fuzzy programming approach. In 2018, Hoseini et al. [8] provided a stochastic model for a carve-out scheduling system to detect the providers optimal schedule. They studied the effect of no-show and double-booking factors on improving waiting time and overtime costs.

Deceuninck et al. [2] also studied the problem of appointment scheduling in the presence of unpunctual patients. Peres et al. [23] also analyzed various appointment scheduling policies in a bariatric surgery clinic in Brazil, considering the following complexity factors: (I) stochastic service times, (II) patient unpunctuality, (III) service interruptions, and (IV) patient no-shows. Alizadeh et al. [1] used a new mixed-integer linear programming model for non-emergency outpatient appointment scheduling with highly demanded medical services considering patient priorities. In 2020, to reduce the negative impact of unpunctual patients such as increasing healthcare costs and reducing provider productivity, Pan et al. [22] presented the appointment scheduling (AS) in the simultaneous presence of unpunctual patients, multiple servers, and no-shows. To determine the appointment schedule for reducing the total cost of waiting time and overtime, they presented a two-stage stochastic mixed-integer programming model.

Table 1 is a summary of studies on improving the appointment scheduling system. This table shows that there is a lack of research in this area that considered the double-book factor as a unique problem. Moreover, there is no fuzzy investigation of service time and unpunctuality considering the show and no-show of the patient. For example, in the study conducted by Moreno and M. Blanco [20], early admission of patients was considered while they reduced hospital stay for the patient. In their study, a fuzzy linear model (fuzzy constraint) was used because of uncertainty in the available amount of clinical services in each time slot. However, in the present study, the parameters of service time and unpunctuality are considered fuzzy due to uncertainty. Therefore, for addressing this lack of research, this paper aims to improve the outpatient appointment scheduling (the time interval between two patients) by minimizing the weighted sum of patients' waiting time, overtime, and idle time in a referral clinic. Besides, both unpunctuality and duration of service factors are considered to have fuzzy values in this research.

Table 1: summary of studies on improving the appointment scheduling system

Authors	Aim to improve	Method	Critical Factors											
			NS	Can.	Lat.	Un-Pun.	Dou.-Book	Ov.-Book	STV	Dis.	Seq.	Pref.	Prio.	IWT
Green & Savin [7]	IWT	QT	*							*		*		*
Muthuraman & Lawley [21]	EP-WTOT	Heu.	*					*	*		*	*		
Robinson & Chen [24]	WTOTIT	DP	*						*					
Zeng et al. [33]	EP-WTOT	LS	*					*			*			
Liu et al. [17]	ENR	Heu.	*							*	*			*
Wang & Gupta [29]	EP	DP, Heu.	*									*		
Lin et al. [16]	WTOT	MDP, ADP	*					*			*			
Erdogan & Denton [4]	WTOT	SLP	*				*		*		*			
Klassen & Yooga-ligham [11]	WTOTIT	Sim.			*					*				
Feldman et al. [6]	ENP	MDP, Heu.	*							*	*	*		
Zacharias & Pinedo [32]	WTOTIT	Sim., Heu.	*					*	*					
Yan & Tang [31]	EP-WTOTIT	Heu.	*						*		*	*		
Erdogan et al. [5]	WTOT	SIP							*		*			*
Jiang et al. [10]	WTOTIT	SLP	*			*			*					*
Luo et al. [18]	TC (WTITRE)	QT	*											
Zhu et al. (2017)	WTIT	Sim., Heu.				*			*					
Deceuninck et al. (2017)	WTOTIT	SLP	*		*	*			*					
Dogru & Melouk [3]	IDWTOTIT	Sim., Heu.	*		*			*	*		*	*		*
Srinivas & Ravi Ravindran [26]	WTREOTOFTIT	Sim., ML	*					*	*		*			
Li et al. [14]	ES	MDP, Sim.									*	*		
Wang et al. [28]	ER	DP, Sim.										*		
Moreno and M. Blanco [20]	ADLS	ILP, Fuzzy				*			*				*	
Hoseini et al. [8]	WIOT	SP	*				*							
Deceuninck et al. [2]	WTITOT	Sim. Heu.				*			*					

Peres et al.[23]	WTITOT	Sim.	*			*			*	*				
Alizadeh et al. [1]	UDUHAITSD	MILP							*			*	*	
Pan et al. [22]	WTOT	SMIP	*			*							*	
Our study	WTITOT	FLP	*			*	*							

NS: No Show	Dou.book: Double book	Seq.: Sequential
Can.: Cancelled	Ov. -book: Overbooking	Pref.:Preferences
Lat.: Lateness	STV:Service Time Variability	IWT: Indirect Waiting Time
Un-Pun.: Unpunctuality	Dis.: Disrupt	Pa.: Patients
QT: Queuing Theory	Heu.:Heuristic	DP: Dynamic Programming
LS: Local Search	MDP: Markov Decision Process	ADP: Approximate Dynamic Programming
SLP: Stochastic Linear Program	Sim.:Simulation	SIP:Stochastic Integer Programming
SMIP: Stochastic Mixed-Integer Programming		ML: Machine Learning
FLP: Fuzzy Linear Programming	ENP: Expected Net Profit	ENR:Expected Net Reward
EP: Expected Profit	WT: Waiting Time	WTOTIT: Waiting Time, Overtime, Idle Time
WTOT: Waiting Time and Overtime		
IDWTITOT: Indirect Direct Waiting Time, Overtime, Idle, Time		ES: Expected Satisfaction
WTITRE: Waiting time, Idle time, Patient Rejection		ER:Expected Revenue
WTOTWAC: Waiting time, Overtime, Waiting Area Congestion		ILP: integer linear programming
WTREOTOFTIT: Waiting time, Patient Rejection, overtime, overflow time, Idle time		
ADLS: Admission Dates, Length of stay		Prio.: Priorities
MILP: Mixed-Integer Linear Programming		
UDUHAITSD: Undesirable Day, Undesirable Hour Assignment, Idle Time, Switching Doctors		

3 Fuzzy programming model

Modeling processes in the real world are usually not accurate. It is not usually possible to model reality with uncertainty as it is, and there are limitations in modeling. Since reality is not modeled correctly, the modeling result also does not have the necessary performance as a solution for the real world.

Fuzzy optimization is considered in three ways:

1. Assume fuzzy model parameters and coefficients.

2. Do not seek to minimize or maximize the objective function with absolute certainty.

3. Constraints are in fuzzy mode.

Current research is based on Jiangs model [10]. However, in the current study, unpunctuality and service duration parameters have been considered fuzzy. According to the conditions and purpose, the first type of fuzzy optimization is considered in this study (assume fuzzy model parameters and coefficients).

Consider one physician serving patients with appointments. Define N to be the number of patients who made appointments using the web or in person. Define P_{ns} to be the no-show probability of a patient, which can be computed from historical data.

Define T to be the length of a standard session. The physician is supposed to be punctual. The cost ratios for physician's idle time (γ) and overtime (β) are standardized as 1 and 1.5 respectively. Let α_k be the cost of the kth waiting patient. X_k as a decision variable is the interval between the k^{th} and $k+1^{th}$ appointment times.

Variables and parameters indicating a point in time and a period of time:

W_k : the waiting time of the k^{th} patient measured from the time of his/her appointment

V_k : the waiting time of the k^{th} patient measured from the time of his/her arrival

I_k : the idle time of the physician waiting for the k^{th} patient

O: the overtime of the physician

A_k : if k^{th} patient is shown, the value is one, otherwise the value is zero

x_k : the interval between the k^{th} and the $k+1^{th}$ appointments
$\tilde{S}_N$: approximate length of service to the k^{th} patient
$\tilde{U}_k$: approximate unpunctuality of the k^{th} patient
$\theta_{u,k}$: cut for unpunctuality of the k^{th} patient
$\theta_{s,k}$: cut for service time of the k^{th} patient
q_k : maximum tolerance for service time of the k^{th} patient
p_k : maximum tolerance for unpunctuality of the k^{th} patient

This paper purposes to improve outpatient appointment scheduling in a referral clinic by reducing the total weight of patient waiting times, physician idle times, and physician overtime. It can be formulated as a fuzzy programming model as follows:

$$\min z = \sum_{k=1}^{N} (\alpha_k . W_k + \gamma_k . I_k) + \beta . O \tag{1}$$

$$V_1 \geq \quad -\widetilde{U_1} \tag{2}$$

$$V_k \geq V_{k-1} + \tilde{S}_{k-1} - X_{k-1} + \tilde{U}_{k-1} - \tilde{U}_k \qquad k = 2, 3, \ldots, N \tag{3}$$

$$I_1 \geq \tilde{U}_1 \tag{4}$$

$$I_k \geq X_{k-1} - \tilde{U}_{k-1} + \tilde{U}_k - V_{k-1} - \tilde{S}_{k-1} \qquad k = 2, 3, \ldots, N \tag{5}$$

$$W_k \geq V_k - \max\left(0, \left(-\tilde{U}_k\right)\right) \qquad k = 1, 2, \ldots, N \tag{6}$$

$$O \geq V_N + \tilde{S}_N - T + \sum_{k=1}^{N-1} X_k + \tilde{U}_N \tag{7}$$

$$O \geq 0 \tag{8}$$

$$V_k, W_k, I_k \geq 0 \qquad k = 1, 2, \ldots, N \tag{9}$$

Model constraints are as follows:
Constraint (2): Minimum waiting time for the first patient
Constraint (3): Minimum waiting time of second to n^{th} patient
Constraint (4): Minimum idle time of the physician waiting for the first patient
Constraint (5): Minimum idle time of the physician waiting for the second to n^{th} patient
Constraint (6): Minimum waiting time of the N patient measured from the time of his/her appointment
Constraint (7): Minimum overtime of the physician
The constraints 8 and 9 describe system performances in terms of positive variables.
In the following section, different models with various assumptions have been explained which Verdegay's approach [13] has been used to solve the models and convert the fuzzy parameters to linear parameters.

4 Linear programming model with fuzzy parameters (Verdegay's approach)

Consider the following general model:

$$\max z = CX \tag{10}$$

$$(AX)_i \leq \widetilde{b_i}, \qquad i = 1,\ 2,\ \ldots,\ m \tag{11}$$

$$X \geq 0 \tag{12}$$

Define $\tilde{b}_i$ to be a maximum source constraint.
In this study, verdegay's approach is used for non-fuzzy the model which is written as follows:

$$\max CX \tag{13}$$

$$(AX)_i \leq b_i + (1-\alpha)\, p_i,\ \forall i \tag{14}$$

$$X \geq 0\ ,\ \alpha \in [0,1] \tag{15}$$

α : level cut
b_i : Fuzzy parameter definite value
P_i : maximum tolerances
In this paper, four different models have been developed based on the show, no-show of patients, single-book, and double-book as follows:

4.1 Model 1 (show of patients - the interval between the k^{th} and the $k+1^{th}$ appointments is constant)

The purpose of model 1 is to reduce the total weight of waiting time, idle time, and overtime. The assumptions of this model are the show of all patients and the constant interval between the k^{th} and the $k+1^{th}$ appointments. In other words, in this model, we suppose all patients are shown by a physician on time and the probability of no show is zero. By solving this model and considering the fuzzy parameters of unpunctuality and service duration, the optimal interval of two consecutive shifts is obtained.

$$\min Z = \sum_{k=1}^{N} (\alpha_k.W_k + \gamma_k.I_k) + \beta.O \tag{16}$$

$$\mathrm{V_1\ \geq\ -\ [U_1 +\ (1-\ \theta_{u,1})\ .\ p_1]} \tag{17}$$

$$\begin{aligned} V_k \geq V_{k-1}\ +\ [S_{k-1} +\ (1\ -\ \theta_{S,k-1})\,.\,q_{k-1}]\ X + [U_{k-1} +\ (1-\ \theta_{u,k-1})\ .\ p_{k-1}] \\ [U_k +\ (1-\ \theta_{u,k})\,.p_k] \qquad\qquad k\ =\ 2,3,\ldots,N \end{aligned} \tag{18}$$

$$I_1 \geq\ [U_1 +\ (1-\ \theta_{u,1})\,.\ P_1] \tag{19}$$

$$\begin{aligned} I_k \geq\ X\ \ [U_{k-1} +\ (1-\ \theta_{u,k-1})\,.\ p_{k-1}] +\ [U_k +\ (1-\ \theta_{u,k})\,.p_k]\,V_{k-1} - [S_{k-1} \\ +\ (1-\ \theta s, k-1)\ .\ qk-1] \qquad\qquad k\ =\ 2,\ 3\ldots,N \end{aligned} \tag{20}$$

$$W_k \geq V_k\ \ max\ (0,\ -\ [U_k\ +\ (1-\ \theta_{u,k})\ .\ p_k]) \qquad\qquad k\ =\ 1,2,\ldots,N \tag{21}$$

$$O\ \geq\ V_N\ +\ [S_N\ +\ (1-\ \theta_{S,N})\ .\ q_N\,]\ \ T\ +\ (N-1)\,.X\ +[U_N + (1-\theta_{u,N})\,.p_N] \tag{22}$$

$$O\ \geq\ 0 \tag{23}$$

$$V_k, W_k, I_k \geq\ 0 \qquad\qquad k\ =\ 1,\ 2\ldots,\ N \tag{24}$$

4.2 Model 2 (no-show of patients - the interval between the k^{th} and the $k+1^{th}$ appointments is constant)

The purpose of model 2 is to reduce the total weight of waiting time, idle time, and overtime. The assumptions of this model are the possibility of the no-show of patients (20%) and the constant interval between the k^{th} and the $k+1^{th}$ appointments.
By solving this model and considering the fuzzy parameters of unpunctuality and service duration, the optimal interval of two consecutive shifts is obtained.

$$\min z = \sum_{k=1}^{N} (\alpha_k.A_k.W_k + \gamma_k.I_k) + \beta.O \tag{25}$$

$$V_1 \geq \; - \; [U_1 \; + \; (1 - \; \theta_{u,1}) \; .p_1 \;] \tag{26}$$

$$V_k \geq \; V_{k-1} \; + \; (A_{k-1}). \; [S_{k-1} \; + \; (1 \; - \; \theta_{S,k-1}). \; q_{k-1}] \; X \; + [U_{k-1} + (1-\theta_{u,k-1}). \; p_{k-1} - [U_k + (1-\theta_{u,k}) \, .p_k] \qquad k = 2,3,\ldots,N \tag{27}$$

$$I_1 \geq \; [U_1 \; + \; (1 - \; \theta_{u,1}) \; . \; p_1] \tag{28}$$

$$I_k \geq X \quad [U_{k-1} + \; (1 - \; \theta_{u,k-1}). \; p_{k-1}] + \; [U_k + \; (1 - \; \theta_{u,k}). \; p_k] \; V_{k-1} \quad (A_{k-1}). \; [S_{k-1} + (1-\theta_{S,k-1}) \, .q_{k-1} \qquad k = 2,3,\ldots,N \tag{29}$$

$$W_k \geq \; V_k \; max \; (0, \; - \; [U_k \; + \; (1 - \; \theta_{u,k}) \; . \; p_k]) \qquad k \; = \; 1,2,\ldots,N \tag{30}$$

$$O \; \geq \; V_N + \; (A_N) \; . [S_N \; + \; (1 - \; \theta_{S,N}) \; . \; q_N \,] \; T \; + \; + \; (N-1) \, .X \; +[U_N + (1-\theta_{u,N}) \, .p_N \tag{31}$$

$$O \; \geq \; 0 \tag{32}$$

$$V_k, W_k, I_k \geq 0 \qquad k = 1,2,\ldots,N \tag{33}$$

4.3 Model 3 (show of patients double book factor)

The purpose of model 3 is to reduce the total weight of waiting time, idle time, and overtime. The assumptions of this model are the show of all patients and the double book. In other words, in this model, we suppose all patients arrive on time and two patients have the same time to visit the physician.
By solving this model and considering the fuzzy parameters of unpunctuality and service duration, the optimal interval of two consecutive shifts is obtained.

$$\min Z = \sum_{k=1}^{N} (\alpha_k.W_k + \gamma_k.I_k) + \beta.O \tag{34}$$

$$V_1 \; \geq \; - \; [U_1 + \; (1 - \; \theta_{u,1}) \; . \; p_1] \tag{35}$$

$$V_k \; \geq V_{k-1} \; + \; [S_{k-1} + \; (1 \; - \; \theta_{S,k-1}) \, .q_{k-1}] \; X_{k-1} \; + \; [U_{k-1} \; + \; (1 - \; \theta_{u,k-1}). \; p_{k-1}] - [U_k + \; (1 - \; \theta_{u,k}) \; . \; p_k] \qquad k \; = \; 2,3,\ldots,N \tag{36}$$

$$I_1 \; \geq \; [U_1 + \; (1 - \; \theta_{u,1}) \; . \; p_1] \tag{37}$$

$$I_k \geq \; X_{k-1} \quad [U_{k-1} + \; (1 - \; \theta_{u,k-1}) \; . \; p_{k-1}\,] \; + \; [U_k \; + \; (1 - \; \theta_{u,k}) \; . \; p_k] \quad V_{k-1} \quad [S_{k-1} \; + \; (1 - \; \theta_{S,k-1}) \; . \; q_{k-1}] \qquad k = 2,3,\ldots,N \tag{38}$$

$$W_k \geq \; V_k \; max \; (0, \; - \; [U_k + \; (1 - \; \theta_{u,k}) \; . \; p_k]) \qquad k \; = \; 1,2,\ldots,N \tag{39}$$

$$O \geq V_N + [S_N + (1- \theta_{S,N}) . q_N] \ T + \sum_{k=1}^{N-1} X_k + [U_N + (1 - \theta_{u,N}) .p_N] \tag{40}$$

$$O \geq 0 \tag{41}$$

$$V_k, W_k, I_k \geq 0 \qquad k = 1, 2, \ldots, N \tag{42}$$

4.4 Model 4 (no-show of patients - double book factor)

The purpose of model 4 is to reduce the total weight of waiting time, idle time, and overtime. The assumptions of this model are the possibility of the no-show of patients (20%) and double book. In other words, in this model, the patients may not be presented and two patients have the same time to visit the patients.
By solving this model and considering the fuzzy parameters of unpunctuality and service duration, the optimal interval of two consecutive shifts is obtained.

$$\min z = \sum_{k=1}^{N} (\alpha_k.A_k.W_k + \gamma_k.I_k) + \beta.O \tag{43}$$

$$V_1 \geq - [U_1 + (1- \theta_{u,1}) . p_1] \tag{44}$$

$$V_k \geq V_{k-1} + (A_{k-1}) . [S_{k-1} + (1 - \theta_{S,k-1}) .q_{k-1}] \ X_{k-1} + [U_{k-1} + (1 - \theta_{u,k-1}) . p_{k-1} - [U_k + (1 - \theta_{u,k}) .p_k] \qquad k = 2, 3, \ldots, N \tag{45}$$

$$I_1 \geq [U_1 + (1- \theta_{u,1}) . p_1] \tag{46}$$

$$I_k \geq X_{k-1} \quad [U_{k-1} + (1- \theta_{u,k-1}) . p_{k-1}] + [U_k + (1- \theta_{u,k}) . p_k] - V_{k-1} - (A_{k-1}) . S_{k-1} + (1 - \theta_{S,k-1}) .q_{k-1}] \qquad k = 2, 3, \ldots, N \tag{47}$$

$$W_k \geq V_k \ \ max \ (0, - [U_k + (1- \theta_{u,k}) .p_k]) \qquad k = 1, 2, \ldots, N \tag{48}$$

$$O \geq V_N + (A_N) . [S_N + (1- \theta_{S,N}) . q_N] \ T + + \sum_{k=1}^{N-1} X_k + [U_N + (1 - \theta_{u,N}) .p_N] \tag{49}$$

$$O \geq 0 \tag{50}$$

$$V_k, W_k, I_k \geq 0 \qquad k = 1, 2, \ldots, N \tag{51}$$

5 Numerical experiments

The numerical experiment is based on a 4-hour treatment session. In this study, a physician and 30 patients are considered to be modeled. We normalize 10 minutes to a unit of 1, and so, the standard session length is T=24.
The service time is approximately one unit (10 minutes) and patients are assumed to be almost punctual. The maximum tolerance for service time fits a normal distribution with an expected value of $\mu_q = 1$ and a variance of $\sigma_q^2 = 0.2^2$. According to the data recorded at the clinic, the probability of the patient being no-show is 0.2. Rates of idle time, overtime, waiting time are based on data recorded in the clinic and Jiang et al. [10] research.
These ratios were assumed to be 1, 1.5, and 0.1 respectively. The maximum tolerance for punctuality fits a normal distribution with an expected value of $\mu_p = 1$ and a variance of $\sigma_p^2 = 0.25^2$. Then the model is solved based on the following scenarios in Table 2. The fuzzy programming is coded in GAMS 24.9.1.

Table 2: Scenarios based on unpunctuality, service time, and the time interval between two appointments

Scenario	Unpunctuality				Service time			X		
	σ_p	$2\sigma_p$	$3\sigma_p$	$4\sigma_q$	σ_p	$2\sigma_q$	$3\sigma_q$	0.5	1	1.5
1**	*				*			*		
2	*				*				*	
3	*				*					*
4	*					*		*		
5	*					*			*	
6	*					*				*
7	*						*	*		
8	*						*		*	
9	*						*			*
10		*			*			*		
11		*			*				*	
12		*			*					*
13		*				*		*		
14		*				*			*	
15		*				*				*
16		*					*	*		
17		*					*		*	
18		*					*			*
19			*		*			*		
20			*		*				*	
21			*		*					*
22			*			*		*		
23			*			*			*	
24			*			*				*
25			*				*	*		
26			*				*		*	
27			*				*			*
28				*	*			*		
29				*	*				*	
30				*	*					*
31				*		*		*		
32				*		*			*	
33				*		*				*
34				*			*	*		
35				*			*		*	
36				*			*			*

** This means that the maximum tolerance for unpunctuality and for service time is 0.25 and 0.2, respectively. The time interval between two appointments is 0.5 units, which equals 5 minutes. The same is the case with other scenarios.

6 Results

The aim of the implementation of the fuzzy programming model is to improve the outpatient appointment scheduling (the time interval between two patients) in the referral clinic by reducing the total weight of waiting time, idle time, and overtime.

This study was conducted on 30 patients referring to the selected clinic. Table 3 shows scenarios results for four fuzzy models. For example, in second model, the first scenario in which the time interval between two appointments is 0.5

units (5 minutes) has the lowest objective function value by 11.365 among the first three scenarios by 11.365, 13.963, and 32.946 respectively.
The developed models were solved by the Cplex solver of GAMS software version 24.9.1 using a personal computer with Pentium (R) Dual-Core CPU T4300 @ 2.10GHz 2.10 GHz and Memory (RAM) 3.

Table 3: Scenarios results for four fuzzy models

Scenario	Unit	Model 1	Model 2	Model 3	Model 4
1	0.5	25.029	11.365	36.279	20.715
2	1	15.822	13.963	25.779	11.965
3	1.5	33.445	32.946	19.658	9.304
4	0.5	22.709	8.386	33.959	17.736
5	1	13.305	12.548	23.459	9.986
6	1.5	32.935	32.446	15.149	7.019
7	0.5	22.768	9.198	32.49	16.101
8	1	15.972	13.266	22.022	8.494
9	1.5	33.956	33.455	17.522	7.122
10	0.5	24.892	11.284	36.142	20.634
11	1	15.057	13.968	25.642	11.884
12	1.5	33.774	33.311	19.421	9.015
13	0.5	22.572	8.304	33.822	17.654
14	1	12.99	12.559	23.322	8.904
15	1.5	33.305	32.854	14.912	6.921
16	0.5	22.012	8.574	32.353	16.02
17	1	15.568	13.274	21.854	8.324
18	1.5	34.262	33.798	17.285	7.209
19	0.5	24.958	11.34	36.208	20.69
20	1	15.83	13.784	25.708	11.94
21	1.5	33.588	33.108	19.316	8.959
22	0.5	22.638	8.361	33.888	17.711
23	1	12.753	12.309	23.388	8.961
24	1.5	33.103	32.636	14.807	6.718
25	0.5	22.778	9.236	32.419	16.076
26	1	15.402	13.087	21.92	8.232
27	1.5	34.097	33.606	17.18	6.876
28	0.5	25.769	11.969	35.986	20.484
29	1	18.138	14.052	25.486	11.734
30	1.5	34.072	33.588	20.016	8.942
31	0.5	22.416	9.474	33.666	17.505
32	1	14.428	12.574	23.166	8.755
33	1.5	33.535	33.065	16.359	6.846
34	0.5	25.08	11.224	32.197	15.87
35	1	17.469	13.265	23.88	10.124
36	1.5	34.529	34.039	19.367	7.26
Model 1: Probability of no-show equal to zero					
Model 2: Probability of no-show equal to 20%					
Model 3: Probability of no-show equal to zero and with double-book factor					
Model 4: Probability of no-show equal to 20% and with double-book factor					

6.1 Analysis of results

According to Table 3, the analysis of four models results in 36 scenarios is showed in Figure 1. The black columns in Figure 1 show the time interval between two appointments in each scenario in which the secondary axis in the right side of figure shows its value.
For example, in first scenario, the time interval between two appointments is 0.5 units (5 minutes). The four lines in

Figure 1 also represent the objective function values of each the four fuzzy models.
For example, in second model, the first scenario in which the time interval between two appointments is 0.5 units (5 minutes) has the lowest objective function value by 11.365 among the first three scenarios by 11.365, 13.963, and 32.946 respectively.

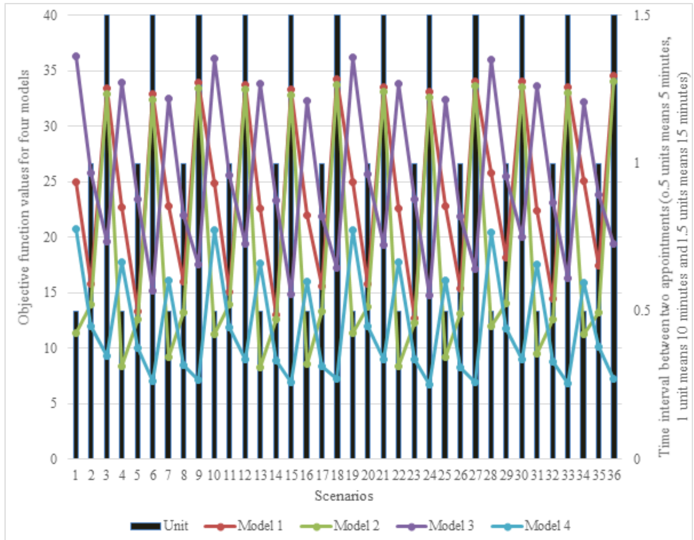

Figure 1: Comparison of scenario results in four models

Based on Table 3 and Figure 1, the analysis of the model results is described as follows:

1. In the first model, scenarios with a time interval between two appointments of 1 unit (10 minutes) have the lowest objective function value among intervals of 0.5 and 1.5 units. Therefore, these kinds of scenarios are the best selected scenarios in the first model. Figure 2 shows the results of the time interval of 1 unit with the selected scenarios written on a graph. This means that scenarios 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, and 35 have the optimal value for the first model (probability of no-show equal to zero). These scenarios were presented in Table 2.

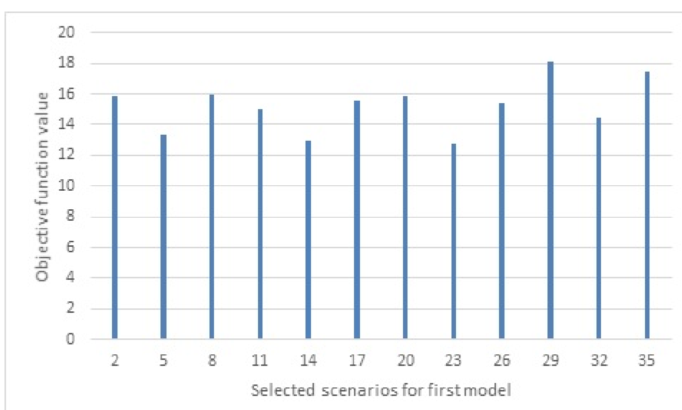

Figure 2: Optimal values of the first model's objective function in the best selected scenarios

2. In the second model, scenarios with a time interval between two appointments of 0.5 units (5 minutes) have the lowest objective function value among intervals of 1 and 1.5 units. Therefore, these kinds of scenarios are the best selected scenarios in the second model. Figure 3 shows the results of the time interval of 0.5 unit with the selected scenarios written on a graph. This means that scenarios 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, and 34 have the optimal value for the second model (probability of no-show equal to 20%). These scenarios were presented in Table 2.

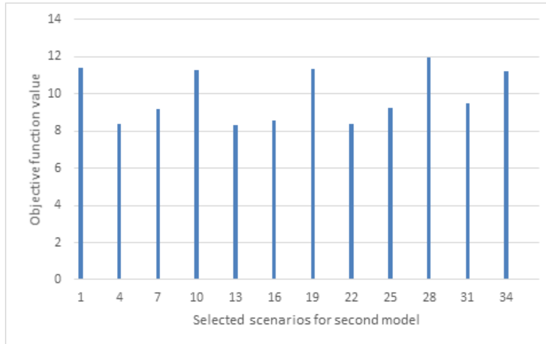

Figure 3: Optimal values of the second models objective function in the best selected scenarios

3. In the third model, scenarios with a time interval between two appointments of 1.5 units (15 minutes) have the lowest objective function value among intervals of 0.5 and 1 units. Therefore, these kinds of scenarios are the best selected scenarios in the third model. Figure 4 shows the results of the time interval of 1.5 units with the selected scenarios written on a graph. This means that scenarios 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, and 36 have the optimal value for the third model (probability of no-show equal to zero and with double-book factor). These scenarios were defined in Table 2.

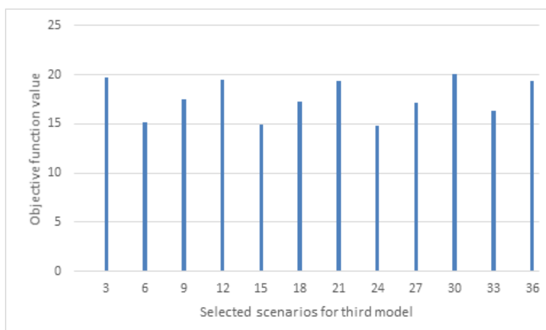

Figure 4: Optimal values of the third models objective function in the best selected scenarios

4. In the fourth model, scenarios with a time interval between two appointments of 1.5 units (15 minutes) have the lowest objective function value among intervals of 0.5 and 1 units. Therefore, these kinds of scenarios are the best selected scenarios in the fourth model. Figure 5 shows the results of the time interval of 1.5 units with the selected scenarios written on a graph. This means that scenarios 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, and 36 have the optimal value for the fourth model (probability of no-show equal to 20% and with double-book factor). These scenarios were defined in Table 2.

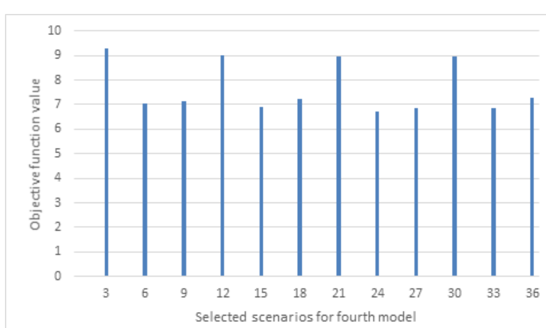

Figure 5: Optimal values of the fourth models objective function in the best selected scenarios

7 Discussions

The purpose of this study was to improve outpatient appointment scheduling in the selected referral clinic to minimize cost in terms of the linear weighted combination of patient waiting times, physician idle times, and physician overtime, using fuzzy linear programming. For this aim, the same scenarios were solved for four models. After solving four models according to the defined scenarios, the best time intervals between two appointments for the first to fourth models were obtained as 1 (10 min), 0.5 (5 min), 1.5 (15 min), and 1.5 (15 min) units, respectively.

In term of considering the waiting time of patients and the utilization of the physician, the current research findings correspond with the results of Jerbi and Kamoun [9], Erdogan and Denton [4], and Jiang et al. [10]. Jerbi and Kamoun [9] minimized the sum of waiting time deviations of the categorized patients and the utilization of the physician. Erdogan and Denton [4] conducted a study to minimize the total weight of patients' waiting time and physician overtime to find the optimal time of arrival for patients to receive medical care. Jiang et al. [10] also minimized the weighted average of patient waiting time, unemployment time, and specialist overtime using the stochastic programming model. Based on their results, the interval between the first and second patients requires more planning which is corresponds with the current research models.

In 2018, Hoseini et al. [8] used a stochastic model for a system to find the main optimal schedule. In addition, they found that no-show and double-booking parameters enhance waiting time and overtime costs which corresponds with the current research findings. Deceuninck et al. [2] also investigated the problem of appointment scheduling in the presence of unpunctual patients and found the negative impact of unpunctual patients on the objectives of appointment scheduling. These results also corresponds with the current research findings. Peres et al. [23] also studied different appointment scheduling approaches in a bariatric surgery clinic in Brazil, considering the following complexity factors: (I) stochastic service times, (II) patient unpunctuality, (III) service interruptions, and (IV) patient no-shows. Their results correspond with the current research findings that patient unpunctuality and patient no-shows need to be considered in appointment scheduling problem.

The purpose of the present study, as described above, was to minimize cost in terms of the linear weighted combination of patient waiting times, physician idle times, and physician overtime. However, in this study, linear programming with fuzzy parameters was used which considered the parameters of service time and unpunctuality in fuzzy. Also, the time intervals between two appointments were considered as parameters in three modes of 5, 10, and 15 minutes. Different assumptions have been considered that include the show of all patients, the no-show of patients, and the double-book factor. The result is the best time intervals between two appointments 5, 10, and 15 minutes in the solved scenarios and models.

8 Conclusions

Due to the importance of waiting time for patients and valuable time for physicians, this study aimed to improve the appointment scheduling system. Consequently, current research minimized the weighted sum of patients' waiting time, overtime, and idle time in a referral clinic. Besides, in this paper, both unpunctuality and duration of service factors are considered to have fuzzy values. The process of investigation was based on the following steps:
1) fuzzy programming of a model,
2) converting to linear programming with Verdegays approach,
3) considering two fuzzy factors,
4) solving four models.

According to the fuzzy programming models, the interval of 10 minutes between two appointments was obtained as the best scenario for the first model. Moreover, according to the results of the second, third, and fourth models, the interval of 5, 15, and 15 minutes between two appointments, respectively, were the best scenarios. Therefore, the selected scenarios in this study are recommended to the selected referral clinic, and they can use these scenarios to improve the performance of the appointment system, taking into account the conditions and rules set by the Ministry of Health.

In other words, for improving appointment systems, clinical mangers need to minimize the waiting time of patients and the idle time of the physician for patients and the overtime of the physician as well as consider the uncertainties in the unpunctuality of patients and service duration.

In future studies, improving the performance of the appointment system in other referral clinics and comparisons with the current study are feasible. Investigating other methods such as simulation and combining under the model used in this study and considering other constraints such as space, number of chairs, and medical facilities are recommended for further research.

All four models of the paper are linear and no variables are binary. So, these models are in the P complexity class and not categorized in the np-hard class. Increasing the size of variables in the P complexity class problem does not increase the solution time as much as in np-hard class problem. When the size of the variables increases in this linear problem (number of patients > 30 and physicians>1 in models of the paper), we can use methods such as lagrangian relaxation to solve the problem in a shorter time. Therefore, modifying these assumptions create opportunities for further research in the future.

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